

Fiber-optic realization of anisotropic depolarizing quantum channels

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We employed an electrically driven polarization controller to implement anisotropic depolarizing quantum channels for the polarization state of single photons. The channels were characterized by means of ancilla-assisted quantum process tomography using polarization-entangled photons generated in the process of spontaneous parametric downconversion. The demonstrated depolarization method offers good repeatability, low cost, and compatibility with fiber-optic setups. It does not perturb the modal structure of single photons, and therefore can be used to verify experimentally protocols for managing decoherence effects based on multiphoton interference. © 2008 Optical Society of America
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1. INTRODUCTION

The practical success of quantum technologies relies on methods to protect and to enhance quantum effects in the presence of decoherence. For example, imperfect entanglement between distant parties can be improved using protocols for entanglement purification and distillation [1]. Transmission and storage of quantum superpositions can be made more robust with the help of quantum error correction techniques [2]. Demonstrating experimentally the feasibility of these methods is a crucial step in the development of quantum technologies.

In this paper we present an experimental realization of a depolarizing fiber optical channel that can be used to simulate a range of nontrivial decoherence mechanisms. We anticipate that the constructed setup can provide a testbed to verify a number of theoretical proposals for managing decoherence effects. As optical methods for generating and manipulating entangled states are relatively well-developed, photonic systems offer a promising route to implement more complex protocols for protecting quantum information in the presence of noise. The optical approach has already been used to verify experimentally decoherence-free subspaces [3,4], use them in elementary quantum computing [5] and quantum cryptography [6] protocols, and to carry out single-qubit purification [7] and quantum error correction [8]. The device presented in this paper can be programmed to simulate specific anisotropic depolarizing channels that appear in practical situations, for which more efficient specialized protection methods may exist [9,10].

One practical method to generate decoherence in optical qubits is to introduce correlations between the measured degree of freedom—typically polarization—and another one, such as the frequency [3] or the wave vector [11], that is not sensed by the measuring apparatus, and from the formal point of view can be considered as the environment. This approach however modifies the modal

structure of the fields. A polarization qubit prepared initially as a superposition of one photon in two modes that differ only by their polarization and are otherwise identical, becomes a complex object that cannot be assigned a single spatiotemporal profile. This has profound implications when such qubits are manipulated using what is at present the most accessible experimental tool of multiphoton interference realized in linear-optics networks. Such interference requires excellent matching of the spatiotemporal modes, and its lack provides distinguishing information that deteriorates interference visibility [12,13]. For demonstrating more complex manipulations of noise-affected states based on multiphoton interference, quantum channels in the polarization degree of freedom need to be implemented without changing the spatiotemporal properties of the fields, and the approach presented in this paper satisfies this requirement. In our case, the effective quantum channel is obtained through a temporal average over suitably chosen polarization transformations. This approach simulates decoherence rather than induces it through an interaction with another microscopic ancilla system, with a completely equivalent outcome.

Our realization of a depolarizing channel is a fiber-optic counterpart of the free-space scheme used in [7] based on Pockels cells. Transformations of the state of polarization (SOP) are implemented with the help of an electrically driven polarization controller composed of magnetic elements that introduce birefringence by squeezing mechanically the fiber [14]. The controller is an inexpensive low-voltage device, compatible with all-fiber setups and characterized by negligible insertion losses. We verify the action of the channel through ancilla-assisted quantum process tomography [15] using maximally entangled photon pairs generated in the process of spontaneous parametric downconversion [16]. Sending one photon through the channel and performing quantum state tomography

on the entire two-photon state yields complete characterization of the channel thanks to the Jamiolkowski isomorphism [17].

We note that scrambling the SOP of optical fields is an important technique in fiber-optic communications; it can reduce the impact of deleterious polarization-dependent effects in practical transmission systems [18–20]. Compared to applications in high bit rate fiber communication systems, requirements set by proof-of-principle demonstrations of quantum protocols are somewhat different. First, shortening the time scale of polarization scrambling is not of primary importance, as typical experiments with entangled photons require long collection times. Therefore mechanical scramblers are presently well-suited for this class of applications. Second, the actual distribution of the SOP after scrambling is not important, as only ensemble averages matter for experiments in which consecutive outputs are manipulated and detected individually.

The structure of this paper is as follows. In Section 2 we summarize the theoretical basis of our experiment. The experimental setup is presented in Section 3, and obtained results are discussed in Section 4. Finally, Section 5 concludes the paper.

2. THEORY

Let us start with a brief theoretical review of qubit channels. The most convenient description is based on the Bloch vector representation. A general channel \mathcal{E} is given by an affine map acting on the Bloch vector \mathbf{r} according to [21],

$$\mathbf{r} \mapsto \mathbf{\Lambda} \mathbf{r} + \mathbf{a}, \quad (1)$$

where $\mathbf{\Lambda}$ is a real 3×3 matrix and \mathbf{a} is a vector characterizing the output state for a completely mixed input. We will be interested here only in unital maps, which do not change the maximally mixed state, and therefore $\mathbf{a} = \mathbf{0}$. Singular value decomposition enables us to represent $\mathbf{\Lambda}$ as a product, $\mathbf{\Lambda} = \mathbf{O} \mathbf{D} \mathbf{O}'$, of two proper rotations \mathbf{O} , \mathbf{O}' and a diagonal matrix $\mathbf{D} = \text{diag}(D_x, D_y, D_z)$. This allows us to separate the trivial rotations of the Bloch vector corresponding to unitary transformations from the depolarization process, described by the three singular values comprising \mathbf{D} . Distinct from the usual convention, we allow here negative singular values, which stems from the requirement that the rotations \mathbf{O} , \mathbf{O}' are proper.

Physical channels must satisfy the condition of complete positivity, which means that a trivial extension of the channel \mathcal{E} to any auxiliary system yields a positive map. For unital maps this condition implies the following constraints on \mathbf{D} [21]:

$$|D_x \pm D_y| \leq |1 \pm D_z|, \quad (2)$$

where the \pm signs on the left- and the right-hand side of the above equation are taken independently. Geometrically, the set of allowed parameters D_x, D_y, D_z forms a tetrahedron shown in Fig. 1, whose vertices are given by the identity \mathbf{I} and 180° rotations $\mathbf{R}_x, \mathbf{R}_y, \mathbf{R}_z$ about axes of the coordinate system in the Bloch representation. These rotations correspond to unitary transformations described

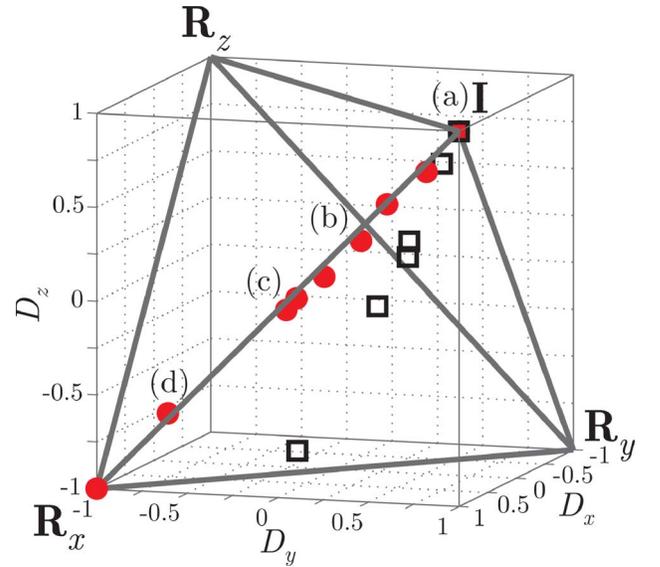


Fig. 1. (Color online) Tetrahedron of singular values for unital quantum channels that results from the physical condition of complete positivity. Dots and squares represent channels implemented and characterized experimentally by means of ancilla-assisted quantum process tomography, which is discussed in detail in Section 4. (a)–(d) refer to channels depicted in Fig. 3.

by Pauli matrices $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$. The geometrical picture shows that the nonunitary depolarizing part of an arbitrary unital channel given by \mathbf{D} can be represented as a convex combination of the identity and three rotations,

$$\mathbf{D} = p_0 \mathbf{I} + p_x \mathbf{R}_x + p_y \mathbf{R}_y + p_z \mathbf{R}_z, \quad (3)$$

where the nonnegative parameters p_0, p_x, p_y, p_z form a probability distribution. The decomposition given in Eq. (3) defines a practical way to implement an arbitrary unital channel by random applications of identity \mathbf{I} and rotations $\mathbf{R}_x, \mathbf{R}_y, \mathbf{R}_z$ with appropriate probabilities, sandwiched between unitary transformations that realize \mathbf{O} and \mathbf{O}' . This approach will be the basis of our experiment.

We will characterize the action of the channel using ancilla-assisted quantum process tomography, which is based on the Jamiolkowski isomorphism [17] between quantum channels and bipartite states. Specifically, preparing two qubits in a maximally entangled bipartite state $|\Phi\rangle$ and sending one of the subsystems through the channel yields a state $\hat{\rho}_\mathcal{E} = (\mathcal{E} \otimes I)(|\Phi\rangle\langle\Phi|)$, that carries complete information about the channel. Performing quantum state tomography on $\hat{\rho}_\mathcal{E}$ allows one to characterize fully channel \mathcal{E} . The density matrix of the maximally entangled input state can be written in the form [22]

$$|\Phi\rangle\langle\Phi| = \frac{1}{4} \left(\hat{1} \otimes \hat{1} - \sum_{i,j=x,y,z} T_{ij} \hat{\sigma}_i \otimes \hat{\sigma}_j \right), \quad (4)$$

where \mathbf{T} is a real 3×3 matrix with the singular values equal to ± 1 . For example, a state $|\Phi_+\rangle = (|\leftrightarrow\rangle + |\uparrow\uparrow\rangle)/\sqrt{2}$, close to the one used in our experiment, corresponds to $\mathbf{T} = \text{diag}(-1, 1, -1)$. For unital channels, the output state $\hat{\rho}_\mathcal{E}$ has a form analogous to Eq. (4) with the matrix \mathbf{T} replaced by $\mathbf{\Lambda} \mathbf{T}$. Its elements can be calculated as expectation values $(\mathbf{\Lambda} \mathbf{T})_{ij} = \text{Tr}[\hat{\rho}_\mathcal{E}(\hat{\sigma}_i \otimes \hat{\sigma}_j)]$. Multiplying the reconstructed matrix by \mathbf{T}^{-1} yields $\mathbf{\Lambda}$. This method tolerates

imperfections in the input state such as mixedness, as long as the matrix \mathbf{T} is invertible. Because our primary interest here is depolarizing properties of quantum channels and unitary rotations can be easily compensated with polarization controllers, we will use experimental data to extract the singular values of Λ that are responsible for the nonunitary dynamics.

It is worth noting that applying the depolarization method described above to a maximally entangled state allows one to generate an arbitrary state of the form given by the right-hand side of Eq. (4). The matrix \mathbf{T} can be brought to the diagonal form analogously to the Bloch representation of unital channels, with the singular values satisfying the analog of Eq. (2), which stems from the positivity condition for density matrices. The output state can be fully controlled through voltages applied to a single device, unlike sources of mixed bipartite states, which rely on more complex preparation procedures [23].

3. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 2. The ultraviolet pump beam entering the setup from the left is prepared by doubling femtosecond pulses from a titanium–sapphire laser (40 fs FWHM pulse duration, 80 MHz repetition rate, 780 nm central wavelength) in a beta-barium borate (BBO) crystal to yield second harmonic with the average power of 14 mW, which is separated from the fundamental with two dichroic mirrors and a blue filter.

The source of entangled photon pairs is based on spontaneous parametric downconversion in a pair of type I BBO crystals oriented at 90° with respect to each other [16]. The half-wave-plate $\lambda/2$ and the Babinet compensator (BC) preceding the crystals ensure that the pairs produced by both crystals are temporally indistinguishable and have equal probability amplitudes. The generated signal and idler photons are sent through red filters (F) and 5 nm bandwidth interference filters (IFs) to filter out the scattered pump field and to restrict the spectral bandwidth.

The idler photons are then coupled into a single-mode fiber using an aspheric lens. They pass first through a manual fiber polarization controller (MPC) with settings fixed at the beginning of the experiment to compensate for the overall SOP change in the fiber, and then through the electrically driven fiber polarization controller (EPC) (OZ-Optics Model EPC-400), which implements the actual

depolarization channel. The idler photons are subsequently coupled out of the fiber into a collimated beam with an aspheric lens and sent through a quarter- and a half-wave-plate and a polarizer to select a specific polarization state, detected finally with a free-space single-photon counting module D1 (PerkinElmer Model SPCM-AQ-131). An identical set of two wave plates and a polarizer is placed in the path of the signal photons before coupling them into a single-mode fiber connected directly to module D2 (PerkinElmer Model SPCM-AQR-14-FC). To reconstruct the density matrix, coincidence count rates were measured on the basis of 16 different two-photon polarization states that form a tomographically complete set. In the standard notation, the basis consisted of the following states: $|\leftrightarrow\rangle$, $|\leftrightarrow\rangle$, $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\circ\leftrightarrow\rangle$, $|\circ\uparrow\rangle$, $|\nearrow\uparrow\rangle$, $|\nearrow\leftrightarrow\rangle$, $|\nearrow\circ\rangle$, $|\nearrow\nearrow\rangle$, $|\circ\nearrow\rangle$, $|\leftrightarrow\nearrow\rangle$, $|\uparrow\nearrow\rangle$, $|\uparrow\circ\rangle$, $|\circ\circ\rangle$, selected by appropriate settings of quarter- and half-wave-plates in both arms. The tomographic reconstruction of the density matrices was carried out using the maximum-likelihood technique [24,25].

The EPC consists of four magnetically driven squeezers driven by external voltages, which apply mechanical stress to the optical fiber thus inducing birefringence. Each squeezer works as a wave plate with a fixed orientation and a tunable phase delay. It is easy to see that placing a sequence of three such squeezers with relative 45° rotations between their optical axes allows one to implement an arbitrary rotation of the Bloch sphere, with phase delays defining the three Euler angles of the rotation [14]. The device used in the experiment included four squeezing elements, and we found that their relative orientation required more complex driving to achieve the necessary polarization rotations. Switching between several different settings of applied voltages and averaging over time yields a depolarizing channel with controllable characteristics. The time of averaging was determined by the brightness of our source of entangled photon pairs and was chosen to be 70 s for each setting of the quarter- and half-wave-plates needed for density matrix reconstruction. That time was much larger compared to the duration of a single setting of the polarization controller, which was of the order of 100 ms (exact figures will be specified in Section 4). By employing a polarization controller faster than the one used in our experiment [20], this depolarization method could be used with much shorter acquisition times and thus with brighter sources. The key advantage of this depolarization method is that it does not introduce correlations between the polarization state and spatial or spectral degrees of freedom of transmitted photons, as long as polarization mode dispersion can be neglected over the spectral range of the transmitted photons.

We define the quantum channel whose characteristics are to be measured as the polarization transformation experienced by the idler photons between the aspheric lenses coupling into and out of the fiber. The action of the EPC was calibrated by coupling into the channel a macroscopic laser beam prepared in three different states of polarization: horizontal, diagonal, or left-circular, and performing a standard SOP measurement on the output using an ellipsometer that detected intensities of horizontal–vertical, diagonal, and circular components on

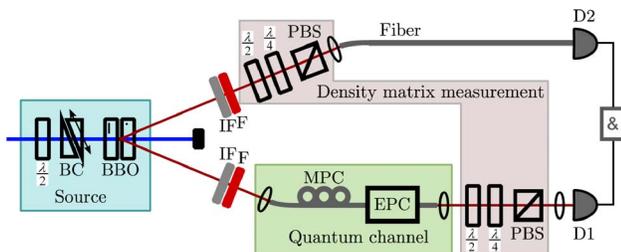


Fig. 2. (Color online) Experimental setup. BC, Babinet compensator; BBO, a pair of beta-barium borate crystals; IF, interference filter; F, red filter; MPC, manual fiber polarization controller; EPC, electrically driven fiber polarization controller; PBS, polarizing beam splitter; D1, D2, single-photon detectors.

three parts split off the input beam. In the following, we will refer to this measurement as simple process tomography to differentiate it from the ancilla-assisted quantum process tomography, that is the core of our experiment. In the first step of the calibration procedure, the driving of the EPC was switched off completely and the MPC was aligned manually such that the action of the channel was equal to the identity. Starting from this reference state, voltage settings required to achieve 180° rotations were determined.

4. RESULTS

The input state for ancilla-assisted quantum process tomography was prepared by setting the action of the quantum channel to identity and measuring the density matrix of the generated state. For measurements described in this paper, coincidence counts were collected over 70 s for each pair of polarization settings. The results of quantum state tomography performed on the input state are shown in Fig. 3(a). The linearized entropy of this state is equal to 0.1 ± 0.1 , and the amount of entanglement can be quantified with the concurrence value of 0.92 ± 0.07 . The input state is well-approximated by a pure state of the form $|\Phi_+\rangle = (|\leftrightarrow\rangle + \exp(i\phi)|\uparrow\uparrow\rangle)/\sqrt{2}$ with the phase equal to $\phi = -0.12$. The fidelity between the measured state and its pure-state approximation is equal to 0.98 ± 0.09 .

We concentrated our efforts on the implementations of anisotropic channels. In the first series of measurements we studied channels that lie on the edge of the tetrahedron, and can be realized as a mixture of the identity \mathbf{I} and a single 180° rotation \mathbf{R} , which is obtained by applying an appropriate voltage to one of the squeezers in the EPC. Thus, the realized quantum channel is given, up to rotations \mathbf{O} and \mathbf{O}' , in the form $\Lambda = (1-p)\mathbf{I} + p\mathbf{R}$. It is easily seen that one of the singular values of such a transformation is equal to 1 irrespective of p , while the remaining two are identical and given by $1-2p$. The voltages driving the EPC were programmed in a temporal loop with a 150 ms period, and the parameter p defined the duty cycle during which the EPC operated as \mathbf{R} .

The experimental results are shown in Figs. 3(b)–3(d). The left column depicts output density matrices reconstructed using the maximum-likelihood algorithm, while the right column represents measured quantum channels with the help of ellipsoids that are obtained by acting with the channels on the input Bloch sphere. It is worth noting that the principal axes of all the ellipsoids are oriented along the same direction to within 3° . This is consistent with our expectations: the transformation \mathbf{R} realized by the EPC is applied along the same direction determined by the orientation of the mechanical squeezer in the EPC, and the birefringence of the input and output fibers, which causes rotations of the SOP, is also fixed.

By means of simple process tomography we found that after taking off the voltage implementing \mathbf{R} the EPC did not return to the state corresponding to the identity channel. We attribute this effect to the residual hysteresis in the magnetic elements of the EPC. We partially compensated it when implementing channels $(1-p)\mathbf{I} + p\mathbf{R}$ by setting a small voltage with an opposite sign during time intervals when the identity transformation \mathbf{I} was to be realized.

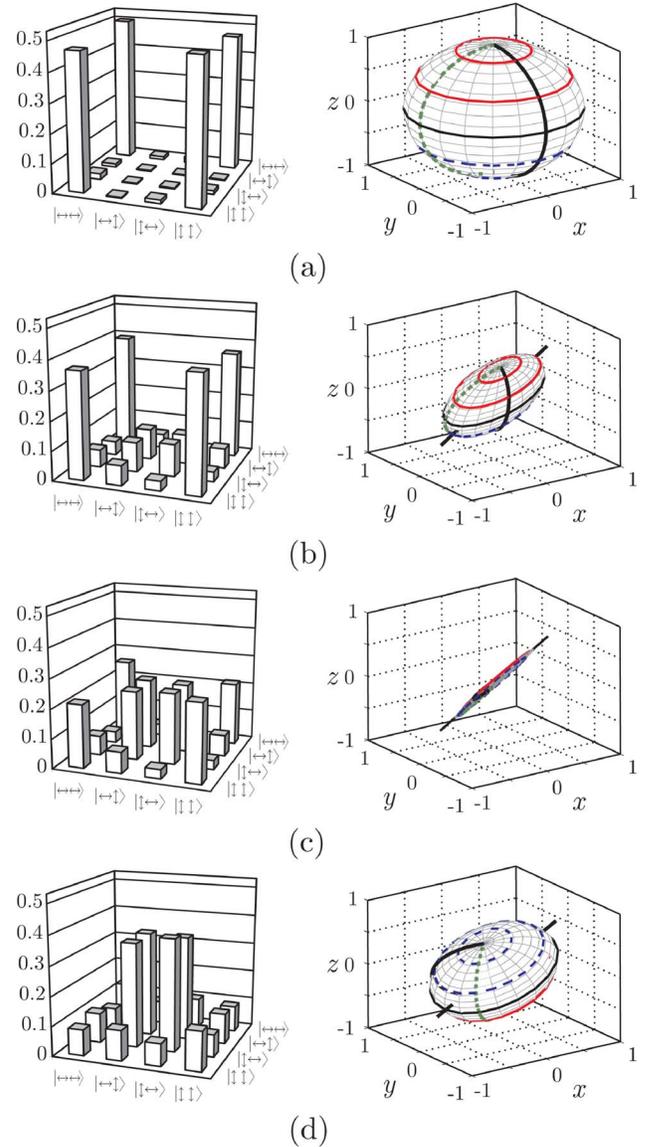


Fig. 3. (Color online) Absolute values of density matrix elements (left) and ellipsoids representing the transformed Bloch sphere (right) measured by ancilla-assisted quantum process tomography for quantum channels obtained by switching between the identity and a selected 180° rotation; (a) the input density matrix and the initial Bloch sphere, with selected parallels and meridians marked for reference; (b)–(d) quantum channels obtained by changing the duty cycle of the voltage driving the controller. Note that the points representing the poles of the original sphere do not coincide with the principal axes of the ellipsoids, marked with straight thick black lines.

The ellipsoids depicted in Fig. 3 contain complete information about quantum channels, including unitary transformations that accompany depolarization. To analyze more closely the depolarizing characteristics of the realized channels, in Fig. 4 we plot the triplets of the singular values versus the programmed duty cycle p , which defines the share of \mathbf{R} in the channel. A good agreement between the predicted and measured singular values is clearly seen.

In the second series of measurements, we implemented quantum channels that are described by mixtures of the

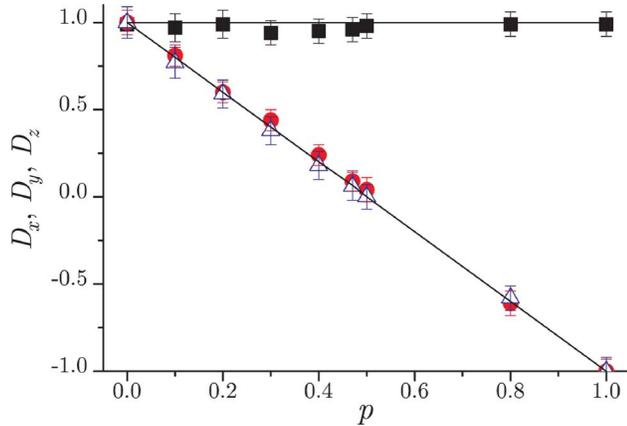


Fig. 4. (Color online) Graph of the singular values for channels of the form $(1-p)\mathbf{I}+p\mathbf{R}$ as a function of the duty cycle p . Each experimentally characterized channel is represented as a triplet of a square, a circle, and a triangle. The solid lines depict values predicted directly from the value of the duty cycle as $(1, 1 - 2p, 1 - 2p)$.

identity and two 180° rotations about orthogonal axes, which we will denote as \mathbf{R}_1 and \mathbf{R}_2 . Such a set of unitary transformations allows one to produce all of the transformations represented by points located on a face of the tetrahedron shown in Fig. 1. Using simple process tomography we searched for voltage settings that would implement such a pair of rotations. The best approximation we were able to find required simultaneous driving of all four squeezers. The angle between the rotation axes was determined to be $86^\circ \pm 3^\circ$, with the rotation angles given by $175^\circ \pm 3^\circ$. These findings strongly suggest that the alignment of the squeezers inside the EPS must have been significantly different from the 45° relative rotations that give parameterization of the polarization transformation in terms of the Euler angles.

We used the rotations \mathbf{R}_1 and \mathbf{R}_2 to realize a series of maps of the form $\Lambda = (1-p)\mathbf{I} + p(\mathbf{R}_1 + \mathbf{R}_2)/2$. Such maps are represented by points located on the median of one of the faces of the tetrahedron depicted in Fig. 1. Analogous to the first series of measurements, the settings of the EPC were updated in a temporal loop with a period of 50 ms. Within a single interval, each of the rotations \mathbf{R}_1 and \mathbf{R}_2 was realized during a fraction $(p/2)$ of the interval, followed by an application of the identity \mathbf{I} , with voltage settings taking into account the partial compensation for the hysteresis.

Exemplary results are presented in Fig. 5 depicting output density matrices and Bloch sphere transformations. To analyze the results more closely, we extracted the singular values and projected them onto the tetrahedron face, shown in Fig. 6. Although the rotations \mathbf{R}_1 and \mathbf{R}_2 were applied over identical fractions of the cycle, the reconstructed maps do not lie on the median of the tetrahedron face. This difference cannot be fully explained by the facts that the rotations \mathbf{R}_1 and \mathbf{R}_2 were characterized by angles deviating from 180° and that their axes were not strictly perpendicular. We attribute the observed disagreement to the hysteresis in the squeezing elements of the EPC. Nevertheless, it is worth noting that within the experimental errors the triplets of the singular values do

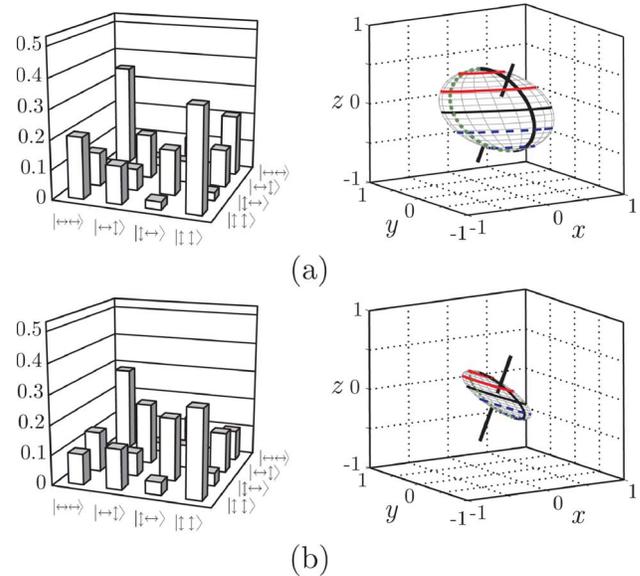


Fig. 5. (Color online) Absolute values of density matrix elements (left) and ellipsoids (right), representing the transformed Bloch sphere shown in Fig. 3(a), measured by ancilla-assisted quantum process tomography for quantum channels obtained by switching between the identity and two 180° rotations with approximately orthogonal axes. Straight thick black lines mark the orientation of the shortest principal axis of an ellipsoid.

form a straight line located on a face of the tetrahedron. The mismatch between the predicted and the observed characteristics of the channel should not pose a substantial difficulty in practice, because the averaged operation of the EPC over the entire period is repeatable to a very good degree. Therefore it is possible to determine, via process tomography, voltage settings that realize a required

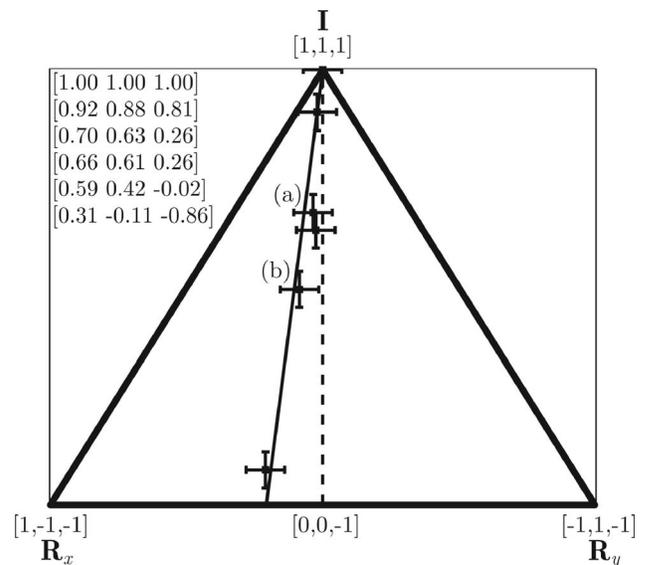


Fig. 6. Orthogonal projection of triplets of singular values for experimentally implemented channels of the form $(1-p)\mathbf{I} + p(\mathbf{R}_1 + \mathbf{R}_2)/2$ onto the tetrahedron face. The solid line is the least-square fit to the experimental points, compared to the median marked with a dashed line. The points labeled (a) and (b) correspond to channels depicted in Fig. 5. The singular values are given explicitly in the upper left corner of the plot.

quantum channel, and use them in further experiments. Moreover, further improvement could be made by using a larger number of squeezing elements, equal to 5 or 6, which, as suggested in [19], should significantly improve the performance of the polarization controller.

5. CONCLUSIONS

We used an electrically driven polarization controller to implement unital quantum channels acting on the polarization state of a single photon. The characteristics of the channels were verified with the help of ancilla-assisted quantum process tomography. Although the controller requires a calibration procedure for voltage settings to implement a given channel, its operation is repeatable to a very good degree. Therefore it can be reliably used in more complex experiment testing methods to manage decoherence effects in emerging quantum-enhanced technologies. Its advantages are low cost, uncomplicated driving, and compatibility with fiber-optic setups.

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